Question 1 — Bivariate Data Analysis **(8 marks)**

(a) Calculate the equation of the least-squares line of best fit for y on x, giving the gradient and the y-intercept correct to one decimal place. (3 marks)

Solution
$\bar{x} = (2+4+6+8+10+12) \div 6 = 7$
$\bar{y} = (3+7+8+13+16+19) \div 6 = 11$
$S_{xy} = \Sigma xy - n\bar{x}\bar{y} = 574 - 6(7)(11) = 112$
$S_{xx} = \Sigma x^2 - n\bar{x}^2 = 364 - 6(7)^2 = 70$
gradient $= S_{xy} \div S_{xx} = 112 \div 70 = 1.6$
y-intercept $= \bar{y} - (\text{gradient})(\bar{x}) = 11 - 1.6(7) = -0.2$
Equation of the line of best fit: $y = 1.6x - 0.2$

Specific behaviours
✓ correct means, $\bar{x} = 7$ and $\bar{y} = 11$
✓ correct gradient, 1.6
✓ correct y-intercept and final equation, $y = 1.6x - 0.2$

(b) Interpret the gradient of this line of best fit in the context of the data. (1 mark)

Solution
For each extra hour of practice completed in a fortnight, a student's test score improvement increases, on average, by 1.6%.

Specific behaviours
✓ correct interpretation of the gradient in context

(c) The correlation coefficient for this data is r = 0.992. Describe the strength and direction of the relationship between hours of practice and score improvement. (1 mark)

Solution
Since r = 0.992 is very close to 1, there is a very strong, positive, linear relationship between hours of practice and score improvement.

Specific behaviours
✓ correct description of strength and direction

(d) Calculate the residual for the student who completed 6 hours of practice. (2 marks)

Solution
Predicted value at x = 6: $y = 1.6(6) - 0.2 = 9.4$
Residual = actual value − predicted value $= 8 - 9.4 = -1.4$

Specific behaviours
✓ correct predicted value, 9.4
✓ correct residual, −1.4

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(e) One student completed 20 hours of practice in a fortnight. Explain why it would not be appropriate to use this line of best fit to predict that student's score improvement. (1 mark)

Solution
20 hours is outside the range of x-values used to calculate the line of best fit (2 to 12 hours), so using the line to predict at x = 20 would be extrapolation, and there is no evidence the linear trend continues beyond the observed data.

Specific behaviours
✓ correct explanation referring to extrapolation beyond the range of the data

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Question 2 — Loans & Investments **(8 marks)**

(a) For Bank A's loan, write a recurrence relation for the balance owing, A_n, after n months, in terms of A_{n-1}, and state the value of A_0. (2 marks)

Solution
Monthly interest rate = 7.2% ÷ 12 = 0.6% = 0.006
$A_0 = 18000$
$A_n = 1.006 \times A_{n-1} - 450$

Specific behaviours
✓ correct monthly interest rate, 0.6%, and $A_0 = 18000$
✓ correct recurrence relation, $A_n = 1.006A_{n-1} - 450$

(b) Use your recurrence relation to calculate the balance owing after 2 months, A_2. (2 marks)

Solution
$A_1 = 1.006 \times 18000 - 450 = 18108.00 - 450 = \$17\,658.00$
$A_2 = 1.006 \times 17658.00 - 450 = 17763.95 - 450 = \$17\,313.95$

Specific behaviours
✓ correct value of A_1, \$17 658.00
✓ correct value of A_2, \$17 313.95

(c) For Bank B's flat-rate loan, calculate the total amount Chevron would repay over the 4 years, and hence his monthly repayment amount. (3 marks)

Solution
Flat interest = P × r × t = 18000 × 0.05 × 4 = \$3600
Total to repay = 18000 + 3600 = \$21 600
Monthly repayment = 21 600 ÷ 48 = \$450

Specific behaviours
✓ correct flat interest amount, \$3600
✓ correct total amount to be repaid, \$21 600
✓ correct monthly repayment, \$450

(d) Chevron notices that Bank A's and Bank B's monthly repayments are both close to \$450. Explain, without further calculation, why Bank A's loan would end up cheaper overall. (1 mark)

Solution
Bank A charges interest only on the reducing (outstanding) balance, so the interest charged each month decreases as the loan is paid off. Bank B charges flat interest on the full original \$18 000 for all 4 years, so the interest amount never decreases. For similar monthly repayments, Bank A will therefore charge less total interest and work out cheaper overall.

Specific behaviours

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✓ correct explanation comparing reducible interest with flat interest over the life of the loan

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Question 3 — Networks & Decision Mathematics **(8 marks)**

(a) Use Kruskal's algorithm to determine a minimum spanning tree connecting all six pump stations. List the edges in the order in which you add them, and give a brief reason for stopping. (4 marks)

Solution
Sort the edges from shortest to longest: CD (90), EF (100), AB (120), DE (140), BC (150), DF (160), BD (180), AC (200), CE (220).
Add CD (90) — no cycle formed.
Add EF (100) — no cycle formed.
Add AB (120) — no cycle formed.
Add DE (140) — connects {C,D} with {E,F}, no cycle formed.
Add BC (150) — connects {A,B} with {C,D,E,F}; all six stations are now connected, so the tree is complete and we stop (5 edges connect 6 vertices).
Minimum spanning tree edges: CD, EF, AB, DE, BC.

Specific behaviours
✓ correctly sorts all edges by length
✓ correctly selects CD, EF and AB as the three shortest edges with no cycle formed
✓ correctly adds DE and then BC to connect all six stations without forming a cycle
✓ correct final set of 5 edges forming a valid spanning tree, with a reason given for stopping

(b) Calculate the total length of pipe required for the minimum spanning tree. (1 mark)

Solution
Total length = 90 + 100 + 120 + 140 + 150 = 600 m

Specific behaviours
✓ correct total length, 600 m

(c) The irrigation company charges \$18.50 per metre to lay pipe. Calculate the total cost of laying the minimum spanning tree network. (2 marks)

Solution
Cost = 600 × \$18.50 = \$11 100

Specific behaviours
✓ correct method, multiplying total length by cost per metre
✓ correct total cost, \$11 100

(d) Explain why the edge D–F, with length 160 m, is not included in the minimum spanning tree. (1 mark)

Solution

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D and F are already connected in the tree via the path D–E–F, so adding the edge DF would create a cycle. Since all six stations are already connected without it, including DF would only add extra length without connecting any new station, so it is excluded from the minimum spanning tree.
Specific behaviours
✓ correct explanation referring to the cycle created and no new station being connected

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