Question 1 — Functions and Graphs **(8 marks)**

Consider the quadratic function $f(x) = -2(x - 1)^2 + 8$.

(a) Describe the sequence of transformations applied to the graph of $y = x^2$ to obtain the graph of $y = f(x)$. (2 marks)

Solution
Dilation by a factor of 2 from the x-axis, together with a reflection in the x-axis (from the −2 coefficient).
Translation of 1 unit in the positive x-direction and 8 units in the positive y-direction.

Specific behaviours
✓ identifies the dilation (factor 2) and reflection in the x-axis
✓ identifies the horizontal translation of 1 unit and the vertical translation of 8 units

(b) Determine the coordinates of the x-intercepts of the graph of $y = f(x)$. (3 marks)

Solution
$-2(x - 1)^2 + 8 = 0$
$(x - 1)^2 = 4$
$x - 1 = \pm 2$
$x = 3$ or $x = -1$
x-intercepts: $(3, 0)$ and $(-1, 0)$

Specific behaviours
✓ forms the correct equation $(x - 1)^2 = 4$
✓ correct values $x - 1 = \pm 2$
✓ correct x-intercepts $(3, 0)$ and $(-1, 0)$

(c) State the range of f. (1 mark)

Solution
The vertex $(1, 8)$ is a maximum, since the parabola opens downward.
Range: $y \leq 8$

Specific behaviours
✓ correct range, $y \leq 8$

(d) Let $g(x) = x + 1$. Determine $f(g(x))$ in simplified form. (2 marks)

Solution
$f(g(x)) = -2((x + 1) - 1)^2 + 8$
$f(g(x)) = -2x^2 + 8$

Specific behaviours
✓ correct substitution of $g(x) = x + 1$ into $f(x)$

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✓ correct simplified expression, $f(g(x)) = -2x^2 + 8$

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Question 2 — Trigonometric Functions **(8 marks)**

Give exact values throughout this question.

(a) Determine the exact value of $\cos(5\pi/6)$. (2 marks)

Solution

$5\pi/6$ lies in the second quadrant, with reference angle $\pi/6$.

$$\cos(5\pi/6) = -\cos(\pi/6) = -\frac{\sqrt{3}}{2}$$

Specific behaviours

✓ identifies the reference angle $\pi/6$ and the correct (second) quadrant

✓ correct exact value, $-\sqrt{3}/2$

(b) Solve $2\sin(x) + 1 = 0$ for $0 \le x \le 2\pi$, giving exact solutions. (3 marks)

Solution

$$2\sin(x) + 1 = 0$$

$$\sin(x) = -\frac{1}{2}$$

Reference angle $\pi/6$; sine is negative in the 3rd and 4th quadrants.

$x = \pi + \pi/6 = 7\pi/6$, or $x = 2\pi - \pi/6 = 11\pi/6$

Specific behaviours

✓ correct equation, $\sin(x) = -1/2$

✓ identifies the reference angle $\pi/6$ and the correct quadrants (3rd and 4th)

✓ correct exact solutions, $x = 7\pi/6$ and $x = 11\pi/6$

(c) State the amplitude and period of $y = 3\cos(2x)$. (2 marks)

Solution

Amplitude = 3

Period = $2\pi/2 = \pi$

Specific behaviours

✓ correct amplitude, 3

✓ correct period, π

(d) State the range of $y = 3\cos(2x) - 1$. (1 mark)

Solution

$3\cos(2x)$ ranges from -3 to 3, so $3\cos(2x) - 1$ ranges from -4 to 2.

Range: $-4 \le y \le 2$

Specific behaviours

✓ correct range, $-4 \le y \le 2$

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Question 3 — Introduction to Differential Calculus **(9 marks)**

Let $f(x) = x^2 + 3x$.

(a) Use first principles to show that $f'(x) = 2x + 3$. (4 marks)

Solution
$f(x + h) = (x + h)^2 + 3(x + h) = x^2 + 2xh + h^2 + 3x + 3h$
$f(x + h) - f(x) = 2xh + h^2 + 3h = h(2x + h + 3)$
$\frac{f(x+h)-f(x)}{h} = 2x + h + 3$
As $h \to 0$, $f'(x) = 2x + 3$

Specific behaviours
✓ correct expansion of $f(x + h)$
✓ correctly simplifies $f(x + h) - f(x)$ and factorises out h
✓ correct difference quotient, $2x + h + 3$
✓ correctly evaluates the limit as $h \to 0$, giving $f'(x) = 2x + 3$

(b) Hence, determine the equation of the tangent to the curve $y = f(x)$ at the point where $x = 1$. (3 marks)

Solution
$f(1) = 1^2 + 3(1) = 4$, so the point is $(1, 4)$
$f'(1) = 2(1) + 3 = 5$
$y - 4 = 5(x - 1) \Longrightarrow y = 5x - 1$

Specific behaviours
✓ correct point, $(1, 4)$
✓ correct gradient, $f'(1) = 5$
✓ correct tangent equation, $y = 5x - 1$

(c) Determine the x-coordinate of the stationary point of the graph of $y = f(x)$. (2 marks)

Solution
$f'(x) = 0$
$2x + 3 = 0$
$x = -\frac{3}{2}$

Specific behaviours
✓ correct equation, $2x + 3 = 0$
✓ correct value, $x = -3/2$

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