Question 1 — Integral Calculus **(8 marks)**

Water flows into a tank at a rate modelled by $V'(t) = 6t - t^2$ (litres per minute), for $0 \le t \le 6$, where V(t) is the volume of water in the tank, in litres, after t minutes. Initially, the tank contains 10 litres of water.

(a) Determine V(t). (3 marks)

(b) Determine the volume of water in the tank after 3 minutes. (2 marks)

(c) Evaluate the definite integral $\int (6t - t^2)\, dt$ from $t = 0$ to $t = 6$, and interpret this value in the context of the tank. (3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

TOP RANKED TUTORING

Question 2 — Discrete Random Variables **(7 marks)**

A discrete random variable X has probability distribution given by $P(X = x) = kx$ for $x = 1, 2, 3, 4$, and $P(X = x) = 0$ otherwise, where k is a positive constant.

(a) Show that $k = 0.1$. (2 marks)

(b) Determine $E(X)$. (2 marks)

(c) Determine $Var(X)$. (3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

TOP RANKED TUTORING

Question 3 — The Normal Distribution **(8 marks)**

The diameters of ball bearings produced by a machine are normally distributed with a mean of 50 mm and a standard deviation of 2 mm. Let X be the diameter, in mm, of a randomly selected ball bearing produced by the machine. You may use the fact that approximately 68% of a normal distribution lies within 1 standard deviation of the mean, 95% lies within 2 standard deviations, and 99.7% lies within 3 standard deviations.

(a) Determine the z-score corresponding to a diameter of 54 mm. (2 marks)

(b) Determine $P(46 < X < 54)$. (2 marks)

(c) Determine $P(X > 52)$. (3 marks)

TOP RANKED TUTORING

(d) The machine rejects any ball bearing with a diameter more than 3 standard deviations from the mean. State the range of diameters that will be accepted. (1 mark)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF