Question 1 — Vectors in the Plane **(7 marks)**

Relative to a fixed origin O, the points A, B and C have position vectors $\mathbf{a} = 3\mathbf{i} + \mathbf{j}$, $\mathbf{b} = 7\mathbf{i} + 7\mathbf{j}$ and $\mathbf{c} = 11\mathbf{i} + 13\mathbf{j}$ respectively.

(a) Find the vectors AB and AC, each expressed in the form $x\mathbf{i} + y\mathbf{j}$. (2 marks)

Solution

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = (7 - 3)\mathbf{i} + (7 - 1)\mathbf{j} = 4\mathbf{i} + 6\mathbf{j}$$

$$\overrightarrow{AC} = \mathbf{c} - \mathbf{a} = (11 - 3)\mathbf{i} + (13 - 1)\mathbf{j} = 8\mathbf{i} + 12\mathbf{j}$$

Specific behaviours

✓ correct vector, $\overrightarrow{AB} = 4\mathbf{i} + 6\mathbf{j}$

✓ correct vector, $\overrightarrow{AC} = 8\mathbf{i} + 12\mathbf{j}$

(b) Hence show that the points A, B and C are collinear. (2 marks)

Solution

$$\overrightarrow{AC} = 8\mathbf{i} + 12\mathbf{j} = 2(4\mathbf{i} + 6\mathbf{j}) = 2\,\overrightarrow{AB}$$

Since AC is a scalar multiple of AB, the vectors AB and AC are parallel. As they share the common point A, the points A, B and C lie on the same straight line, that is, they are collinear.

Specific behaviours

✓ recognises that $\overrightarrow{AC} = 2\,\overrightarrow{AB}$, so AB and AC are parallel

✓ states that, sharing the point A, this makes A, B and C collinear

(c) State the ratio AB : BC. (1 mark)

Solution

$$\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB} = (8\mathbf{i} + 12\mathbf{j}) - (4\mathbf{i} + 6\mathbf{j}) = 4\mathbf{i} + 6\mathbf{j} = \overrightarrow{AB}$$

$$AB : BC = 1 : 1$$

Specific behaviours

✓ correct ratio, AB : BC = 1 : 1

(d) D is the point such that OABD, in that order, is a parallelogram. Find the position vector of D. (2 marks)

Solution

Since OABD is a parallelogram, OD is equal and parallel to AB (opposite sides of a parallelogram are equal and parallel).

$$\overrightarrow{OD} = \overrightarrow{AB} = 4\mathbf{i} + 6\mathbf{j}$$

Position vector of D: $\mathbf{d} = 4\mathbf{i} + 6\mathbf{j}$

Specific behaviours

✓ uses the parallelogram property $\overrightarrow{OD} = \overrightarrow{AB}$

✓ correct position vector, $\mathbf{d} = 4\mathbf{i} + 6\mathbf{j}$

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Question 2 — Matrices **(8 marks)**

Let A be the matrix with rows (2, 1) and (3, 4), that is, A = [[2, 1], [3, 4]].

(a) Find the determinant of A. (1 mark)

Solution

$\det(A) = (2 \times 4) - (1 \times 3) = 8 - 3 = 5$

Specific behaviours

✓ correct determinant, det(A) = 5

(b) Hence write down A^{-1}, the inverse of A. (2 marks)

Solution

For A = [[a, b], [c, d]], $A^{-1} = \frac{1}{\det(A)} \times$ the matrix with rows (d, −b) and (−c, a)

$A^{-1} = \frac{1}{5} \times$ the matrix with rows (4, −1) and (−3, 2), i.e. $A^{-1} = \frac{1}{5}$[[4, −1], [−3, 2]]

Specific behaviours

✓ uses det(A) = 5 to form the scalar factor 1/5

✓ correct inverse matrix with rows (4, −1) and (−3, 2)

(c) The simultaneous equations 2x + y = 7 and 3x + 4y = 18 can be written in matrix form as A[x; y] = [7; 18]. Use A^{-1} to solve this system for x and y. (3 marks)

Solution

$$[x; y] = A^{-1}[7; 18] = \frac{1}{5}[[4, -1], [-3, 2]] \times [7; 18]$$

$$x = \frac{1}{5}(4 \times 7 - 1 \times 18) = \frac{1}{5}(28 - 18) = \frac{10}{5} = 2$$

$$y = \frac{1}{5}(-3 \times 7 + 2 \times 18) = \frac{1}{5}(-21 + 36) = \frac{15}{5} = 3$$

Solution: x = 2, y = 3

Specific behaviours

✓ correctly premultiplies [7; 18] by A^{-1}

✓ correct value, x = 2

✓ correct value, y = 3

(d) Show that the point (1, −1) is invariant under the transformation represented by A, that is, show that A maps (1, −1) to itself. (2 marks)

Solution

A(1, −1): first row gives 2(1) + 1(−1) = 2 − 1 = 1

second row gives 3(1) + 4(−1) = 3 − 4 = −1

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So A maps (1, −1) to (1, −1); since the image equals the original point, (1, −1) is invariant under A.

Specific behaviours
✓ correctly multiplies A by the column (1, −1)
✓ obtains the image (1, −1), confirming the point is invariant

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Question 3 — Geometry and Trigonometry **(7 marks)**

In triangle ABC, AB = 8 cm, AC = 5 cm and angle BAC = 60°.

(a) Use the cosine rule to find the exact length of BC. (3 marks)

Solution

$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos(BAC)$

$BC^2 = 8^2 + 5^2 - 2(8)(5)\cos 60°$

$BC^2 = 64 + 25 - 80 \times \frac{1}{2} = 89 - 40 = 49$

$BC = \sqrt{49} = 7$ cm

Specific behaviours

✓ correct substitution into the cosine rule
✓ correct simplification, $BC^2 = 49$
✓ correct exact length, BC = 7 cm

(b) Show that $\sin(ABC) = \frac{5\sqrt{3}}{14}$. (2 marks)

Solution

By the sine rule, $\frac{\sin(ABC)}{AC} = \frac{\sin(BAC)}{BC}$

$\sin(ABC) = \frac{5\sin 60°}{7}$

$\sin(ABC) = \frac{5 \times \frac{\sqrt{3}}{2}}{7} = \frac{5\sqrt{3}}{14}$

Specific behaviours

✓ correct setup of the sine rule relating angle ABC and angle BAC
✓ correct exact value, $\sin(ABC) = (5\sqrt{3})/14$

(c) Hence, or otherwise, find the exact area of triangle ABC. (2 marks)

Solution

$\text{Area} = \frac{1}{2} \times AB \times AC \times \sin(BAC)$

$\text{Area} = \frac{1}{2} \times 8 \times 5 \times \sin 60° = 20 \times \frac{\sqrt{3}}{2}$

$\text{Area} = 10\sqrt{3}$ cm²

Specific behaviours

✓ correct substitution into the area formula (1/2)ab sin C
✓ correct exact area, $10\sqrt{3}$ cm²

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