Question 1 — Complex Numbers **(8 marks)**

(a) Express $-27i$ in exact polar form. (2 marks)

Solution
$-27i$ lies on the negative imaginary axis, so it has modulus 27 and argument $-\pi/2$.
$-27i = 27\,\text{cis}(-\pi/2)$

Specific behaviours
✓ correct modulus, 27
✓ correct argument, $-\pi/2$

(b) Hence determine, in exact polar form, all three solutions to $z^3 = -27i$. (3 marks)

Solution
$\lvert z\rvert = 27^{1/3} = 3$
$\arg(z) = \dfrac{-\pi/2 + 2k\pi}{3}$, for k = 0, 1, 2
k = 0: arg = $-\pi/6$ k = 1: arg = $\pi/2$ k = 2: arg = $7\pi/6 \equiv -5\pi/6$
$z0 = 3\,\text{cis}(-\pi/6)$, $z1 = 3\,\text{cis}(\pi/2)$, $z2 = 3\,\text{cis}(-5\pi/6)$

Specific behaviours
✓ correct modulus, 3
✓ correct general argument expression, $(-\pi/2 + 2k\pi)/3$
✓ all three correct arguments / solutions stated

(c) One of the solutions found in part (b) is purely imaginary. State this solution in Cartesian form, and verify that it satisfies the original equation. (3 marks)

Solution
$z1 = 3\,\text{cis}(\pi/2)$ is the purely imaginary solution.
$z1 = 3(\cos(\pi/2) + i\sin(\pi/2)) = 3i$
Check: $(3i)^3 = 27i^3 = 27(-i) = -27i$, which matches the original equation.

Specific behaviours
✓ correctly identifies $z1 = 3\,\text{cis}(\pi/2)$ as the purely imaginary solution
✓ correct Cartesian form, 3i
✓ correctly verifies $(3i)^3 = -27i$

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Question 2 — Integration Techniques **(6 marks)**

(a) Express $\frac{5x+1}{(x-1)(x+2)}$ in the form $\frac{A}{x-1}+\frac{B}{x+2}$, stating the values of the constants A and B. (3 marks)

Solution
$5x + 1 = A(x + 2) + B(x - 1)$
Let $x = 1$: $6 = 3A \ \therefore A = 2$
Let $x = -2$: $-9 = -3B \ \therefore B = 3$

Specific behaviours
✓ correctly forms the identity $5x + 1 = A(x + 2) + B(x - 1)$
✓ correct value, $A = 2$
✓ correct value, $B = 3$

(b) Hence determine $\int \frac{5x+1}{(x-1)(x+2)}\,dx$. (3 marks)

Solution
$\int \frac{5x+1}{(x-1)(x+2)}\,dx = \int \left(\frac{2}{x-1}+\frac{3}{x+2}\right)dx$
$= 2\ln\lvert x-1\rvert + 3\ln\lvert x+2\rvert + c$

Specific behaviours
✓ correctly rewrites the integral using the partial fractions from (a)
✓ correctly antidifferentiates both reciprocal terms using the absolute value
✓ includes the constant of integration, c

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Question 3 — Vectors in Three Dimensions **(8 marks)**

(a) Determine the vectors AB (from A to B) and AC (from A to C), each in component form. (2 marks)

Solution
$AB = B - A = (5 - 2, 4 - 0, 1 - 1) = (3, 4, 0)$
$AC = C - A = (6 - 2, -3 - 0, 3 - 1) = (4, -3, 2)$

Specific behaviours
✓ correct vector, AB = (3, 4, 0)
✓ correct vector, AC = (4, −3, 2)

(b) Hence show that angle BAC = 90°. (2 marks)

Solution
$AB \cdot AC = (3)(4) + (4)(-3) + (0)(2) = 12 - 12 + 0 = 0$
Since $AB \cdot AC = 0$ and neither vector is the zero vector, AB is perpendicular to AC, so angle BAC = 90°.

Specific behaviours
✓ correctly calculates $AB \cdot AC = 0$
✓ correctly concludes angle BAC = 90° since the dot product is zero

(c) Determine a vector equation for the line passing through A and B. (2 marks)

Solution
$r = (2, 0, 1) + t(3, 4, 0),\ t \in \mathbb{R}$

Specific behaviours
✓ uses the correct point, A = (2, 0, 1)
✓ uses the correct direction vector, AB = (3, 4, 0)

(d) Determine whether the point D(11, 12, 1) lies on the line found in part (c). Justify your answer. (2 marks)

Solution
Set $(2 + 3t, 4t, 1) = (11, 12, 1)$.
From the x-coordinate: $2 + 3t = 11 \ \therefore t = 3$
From the y-coordinate: $4t = 12 \ \therefore t = 3$ (consistent)
The z-coordinate, 1 = 1, is automatically satisfied.
Since a single value of t = 3 satisfies all three coordinates, D does lie on the line.

Specific behaviours
✓ solves for t from two coordinates and obtains a consistent value, t = 3
✓ correctly checks the third coordinate and concludes D lies on the line

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