Measurement & Geometry

Question 1 **(6 marks)**

A decorative solid is formed by joining a cone of radius 6 cm and height 8 cm to a hemisphere of the same radius 6 cm, so that the cone's circular base and the hemisphere's flat face coincide exactly. The resulting solid has a single pointed vertex at the bottom of the cone and a rounded dome at the top; it has no flat (exposed) faces.

(a) Calculate the slant height of the cone. (1 mark)

Solution

$$l = \sqrt{r^2 + h^2} = \sqrt{6^2 + 8^2} = \sqrt{100}$$

l = 10 cm

Specific behaviours

✓ correct use of Pythagoras' theorem, l = 10 cm

(b) Calculate the volume of the whole solid, correct to 2 decimal places. (3 marks)

Solution

$$\text{Volume of cone} = \tfrac{1}{3}\pi r^2 h = \tfrac{1}{3}\pi(6)^2(8) = 96\pi \text{ cm}^3$$

$$\text{Volume of hemisphere} = \tfrac{2}{3}\pi r^3 = \tfrac{2}{3}\pi(6)^3 = 144\pi \text{ cm}^3$$

Total volume = 96π + 144π = 240π ≈ 753.98 cm³

Specific behaviours

✓ correct volume of cone, 96π cm³

✓ correct volume of hemisphere, 144π cm³

✓ correct total volume, 753.98 cm³ (2 d.p.)

(c) Calculate the total surface area of the whole solid (the curved surface area of the cone plus the curved surface area of the hemisphere), correct to 2 decimal places. (2 marks)

Solution

Curved surface area of cone = πrl = π(6)(10) = 60π cm²

Curved surface area of hemisphere = 2πr² = 2π(6)² = 72π cm²

Total surface area = 60π + 72π = 132π ≈ 414.69 cm²

Specific behaviours

✓ correct curved surface area of the cone, 60π cm²

✓ correct curved surface area of the hemisphere and correct total, 414.69 cm² (2 d.p.)

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Question 2 **(6 marks)**

In triangle ABC, the angle at A is 35°, the angle at B is 82°, and the side AB = 9.5 m.

(a) Find the size of angle C. (1 mark)

Solution
Angle C = 180° − 35° − 82°
Angle C = 63°
Specific behaviours
✓ correct angle, C = 63°

(b) Use the sine rule to find the length of side BC, correct to 2 decimal places. (3 marks)

Solution
BC / sin A = AB / sin C
BC = (9.5 × sin 35°) / sin 63°
BC = (9.5 × 0.5736) / 0.8910
BC ≈ 6.12 m
Specific behaviours
✓ correct sine rule set-up, BC/sin A = AB/sin C
✓ correct substitution of values
✓ correct final length, BC ≈ 6.12 m (2 d.p.)

(c) Hence find the area of triangle ABC, correct to 2 decimal places. (2 marks)

Solution
$\text{Area} = \frac{1}{2} \times AB \times BC \times \sin B$
Area = 0.5 × 9.5 × 6.12 × sin 82°
Area ≈ 28.77 m^2
Specific behaviours
✓ correct area formula using AB, BC and the included angle B
✓ correct final area, 28.77 m^2 (2 d.p.)

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Number & Algebra

Question 3 **(5 marks)**

(a) Factorise and hence solve: $x^2 - 5x - 24 = 0$. (2 marks)

Solution
$x^2 - 5x - 24 = 0$
$(x - 8)(x + 3) = 0$
$x = 8$ or $x = -3$

Specific behaviours
✓ correct factorisation, $(x - 8)(x + 3)$
✓ correct solutions, $x = 8$ or $x = -3$

(b) Solve $2x^2 + 3x - 7 = 0$ using the quadratic formula, giving your answers correct to 2 decimal places. (3 marks)

Solution
$a = 2, b = 3, c = -7$
Discriminant $= b^2 - 4ac = 3^2 - 4(2)(-7) = 9 + 56 = 65$
$x = \frac{-3 \pm \sqrt{65}}{4}$
$x = \frac{-3 + 8.06}{4} \approx 1.27$ or $x = \frac{-3 - 8.06}{4} \approx -2.77$

Specific behaviours
✓ correct substitution into the quadratic formula and discriminant = 65
✓ correct solution, $x \approx 1.27$
✓ correct solution, $x \approx -2.77$

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Question 4 **(8 marks)**

(a) Evaluate $8^{-2/3}$, showing your working. (2 marks)

Solution
$8^{1/3} = 2$
$8^{-2/3} = (8^{1/3})^{-2} = 2^{-2}$
$8^{-2/3} = \frac{1}{4}$

Specific behaviours
✓ correctly evaluates $8^{1/3} = 2$
✓ correct final value, $8^{-2/3} = 1/4$

(b) Simplify $(m^4n^{-3}) \div (m^{-1}n^2)$, giving your answer with positive indices only. (2 marks)

Solution
$(m^4n^{-3}) \div (m^{-1}n^2) = m^{4-(-1)}n^{-3-2}$
$= m^5n^{-5}$
$= \frac{m^5}{n^5}$

Specific behaviours
✓ correct index for m, m^5
✓ correct index for n and positive-index form, m^5/n^5

(c) Simplify $\sqrt{45} + \sqrt{20}$, giving your answer as a single surd in simplest form. Then, using a similar idea, rationalise the denominator of $\frac{6}{\sqrt{3}}$, giving your answer in simplest surd form. (4 marks)

Solution
$\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$, and $\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$
$\sqrt{45} + \sqrt{20} = 3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$
$\frac{6}{\sqrt{3}} = \frac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{6\sqrt{3}}{3}$
$\frac{6}{\sqrt{3}} = 2\sqrt{3}$

Specific behaviours
✓ correctly simplifies $\sqrt{45} = 3\sqrt{5}$ and $\sqrt{20} = 2\sqrt{5}$
✓ correct final surd, $5\sqrt{5}$
✓ multiplies numerator and denominator by $\sqrt{3}$
✓ correct rationalised answer, $2\sqrt{3}$

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Statistics & Probability

Question 5 **(5 marks)**

The table below shows the number of hours of exercise per week (x) and the resting heart rate in beats per minute (y) for 7 adults. x (hours): 1, 2, 3, 4, 5, 6, 7 y (bpm): 78, 74, 70, 68, 64, 60, 58

(a) State whether the correlation between exercise hours and resting heart rate is positive or negative. (1 mark)

Solution
As the hours of exercise increase, the resting heart rate consistently decreases.
The correlation is negative.
Specific behaviours
✓ correctly identifies a negative correlation

(b) Describe the strength of this correlation, and justify your answer using the data. (2 marks)

Solution
The heart rate decreases by a similar amount (about 2–4 bpm) for every extra hour of exercise, with no outliers.
The points would lie very close to a straight line, so the correlation is strong (strong negative correlation).
Specific behaviours
✓ identifies the correlation as strong
✓ justifies using the consistent, close-to-linear pattern in the data

(c) A line of best fit for this data is given by $y = -3.2x + 81$. Use this equation to estimate the resting heart rate of a person who exercises for 10 hours a week, and state whether this estimate is an example of interpolation or extrapolation. (2 marks)

Solution
$y = -3.2(10) + 81 = -32 + 81$
$y = 49$ bpm
Since $x = 10$ is outside the given data range (1 to 7 hours), this is extrapolation.
Specific behaviours
✓ correct estimate, 49 bpm
✓ correctly identifies this as extrapolation, since $x = 10$ is outside the data range

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Question 6 **(9 marks)**

The number of goals scored by a basketball team in its last 5 games was: 2, 4, 6, 8, 10.

(a) Calculate the mean and the standard deviation of this data set, giving the standard deviation correct to 2 decimal places. (4 marks)

Solution

$$\text{Mean} = \frac{2+4+6+8+10}{5} = \frac{30}{5} = 6$$

Deviations from the mean: −4, −2, 0, 2, 4

Squared deviations: 16, 4, 0, 4, 16; sum = 40

$$\text{Variance} = \frac{40}{5} = 8, \text{ so standard deviation} = \sqrt{8} \approx 2.83$$

Specific behaviours

✓ correct mean, 6
✓ correct deviations and squared deviations
✓ correct sum of squares = 40 and variance = 8
✓ correct standard deviation, 2.83 (2 d.p.)

A bag contains 5 red marbles and 3 blue marbles. A marble is drawn at random, its colour is recorded, and it is then returned to the bag before a second marble is drawn (the two draws are independent).

(b) Using a tree diagram or otherwise, find the probability that both marbles drawn are red. (2 marks)

Solution

Since the marble is replaced, P(red) = $\frac{5}{8}$ on each draw, and the draws are independent.

$$P(\text{both red}) = \frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$$

Specific behaviours

✓ correctly identifies P(red) = 5/8 for each independent draw
✓ correct final probability, 25/64

Suppose instead that the first marble is NOT returned to the bag before the second marble is drawn (the draws are now dependent).

(c) Find the probability that the first marble drawn is red and the second marble drawn is blue. (3 marks)

Solution

P(first marble red) = $\frac{5}{8}$

After removing one red marble (not replaced), 7 marbles remain: 4 red, 3 blue

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P(second marble blue | first red) = $\frac{3}{7}$
$P(\text{red then blue}) = \frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$

Specific behaviours
✓ correct P(first red) = 5/8
✓ correct conditional probability, P(second blue) = 3/7 given no replacement
✓ correct final probability, 15/56

TOP RANKED TUTORING

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